Autonomous GNC Algorithms for Rendezvous Missions to Near-Earth-Objects

Jesús Gil-Fernández*, Raúl Cadenas-Gorgojo†, Tomás Prieto-Llanos‡, Mariella Graziano§
GMV S.A., Tres Cantos, Madrid, Spain

and

Remi Drai**
ESA/ESTEC, Noordwijk, The Netherlands

This paper presents the design of an autonomous GNC system for the rendezvous phase of low-cost missions to very small, faint Near Earth Object (NEO). The optical navigation problem presents a strong relation between the approach strategy and the best combination of sensors, algorithms and actuators. First of all the problem is globally analyzed and the rendezvous strategy defined, from detection of the asteroid to insertion into a desired bounded orbit, considering the observability of the full spacecraft state relative to the asteroid. Then, for each sub-phase the GNC algorithms, including the Image Processing, are described for different propulsion systems (impulsive and low-thrust), and including or not an altimeter. Monte Carlo simulations for different NEO (Apophis and 2003 SM84) are run to tune the algorithms and to assess the system performances. The simulation tool includes high-fidelity models of the optical sensors (navigation camera and star-tracker). The conclusions show the driving parameters and the feasibility of the proposed GNC system.

Nomenclature

\[ \mu = \text{mean of a random variable distribution} \]
\[ \sigma = \text{standard deviation of a random variable distribution} \]
\[ N = \text{dimension of the augmented state vectors in the navigation filter} \]
\[ \mathbf{v} = \text{velocity} \]
\[ v = \text{speed} \]
\[ b = \text{impact parameter} \]
\[ t_{GO} = \text{time-to-go (time to closest approach)} \]
\[ \mathbf{LOS} = \text{unit vector from the SC to the asteroid} \]
\[ \theta = \text{angle between the LOS and the relative velocity} \]
\[ \dot{\theta} = \text{time derivative of variable } \theta \]

I. Introduction

RENDZVOUS missions to asteroids and comets have recently gained a leading role (NEAR, ROSETTA, Hayabusa, Dawn) because the scientific return is higher than in a typical high-velocity fly-by. Several

* Project Manager, Advanced Space Systems & Technologies business unit, Isaac Newton 11, Tres Cantos, 28760 Madrid (Spain), AIAA Member.
† Project Engineer, Advanced Space Systems & Technologies business unit, Isaac Newton 11, Tres Cantos, 28760 Madrid (Spain).
‡ Consultant, Advanced Space Systems & Technologies business unit, Isaac Newton 11, Tres Cantos, 28760 Madrid (Spain), and AIAA Member.
§ Head of Advanced Space Systems & Technologies business unit, Isaac Newton 11, Tres Cantos, 28760 Madrid (Spain).
** Technical Officer, GNC section, Keplerlaan 1, PO Box 299, 2200 AG Noordwijk (The Netherlands).
rendezvous missions to small bodies are currently under investigation because these objects will provide information on the origin and evolution of the Solar System and because accurate characterization of Near Earth Objects (NEO) is required for proper design of a mitigation strategy in case of risk of collision with the Earth.

The rendezvous phase begins with the detection of the asteroid against a starry background, typical initial conditions are 1e6 km distance and several 100 m/s relative velocity, and finishes with the insertion of the spacecraft (SC) into the desired initial orbit around the NEO, typical orbit altitude is several asteroid radii. This phase presents critical operations, such as the orbit insertion with high risk of mission loss, and many complexities such as the great uncertainties in the asteroid characteristics (e.g. mass, shape).

For future low-cost missions to NEO, several technologies are available to decrease the total budget, and an autonomous Guidance, Navigation & Control (GNC) system, including the Image Processing (IP), is one of the most promising. The techniques investigated for autonomous rendezvous with small bodies can be also applied if the mission is not fully autonomous, for instance for on-board check (e.g. Failure Detection, Isolation and Recovery algorithms), to reduce the data transmission in ground-based navigation (part of the IP can be done on-board), or for extended autonomy periods because of communication constraints.

The design of an autonomous rendezvous GNC system for a low-cost mission to a very small asteroid is extremely challenging. Initially the target is very faint and must be detected and tracked against a starry background. Then, the approach strategy must assure the observability of the full relative state vector using only optical measurements. Finally, the SC must be delivered into a safe and stable orbit around the asteroid with the required accuracy, considering the uncertainties in the environment.

The design of the GNC system for autonomous rendezvous with small objects must include the definition of the approach trajectory, the characterization of sensors and actuators, and the selection of the best algorithms for IP and GNC. The particularities of the problem cause that simple trade-off based on qualitative analysis can produce misleading results because the performances are not correctly calculated. Dedicated simulations in a representative environment are required to validate the performances of the GNC system and to ensure the success of the selected mission architecture.

The main requirement on the GNC system is to insert autonomously the SC in the desired orbit around the orbit with a injection error that assures the lack of collision while minimizing the amount of required propellant. From this top level requirement derives other more specific such as the nearly continuous tracking of the asteroid by the IP to provide line-of-sight (LOS) measurements and the achievement of accurate SC state estimation relative to the target so that the guidance and control can compute and execute the necessary maneuvers to cancel possible deviations in the nominal trajectory.

For better analysis and design of the rendezvous problem, two sub-phases are distinguished: far and close approach. During the far approach sub-phase the asteroid is a punctual object in the detector array and most of the time is a very faint object with high variability of the brightness, due to the rotation and irregular shape of the asteroid and the phase angle (Sun-NEO-SC) being close to 90º (see upper right graph of Fig.1 where the higher phase angle the higher the variability of the brightness curve). In addition, in the far approach the relative velocity is high (several hundreds m/s) and the relative state is poorly observable (non-observable if the SC is in course of collision) and some techniques are required to estimate all the components of the relative position and velocity accurately enough.

In the close approach, the asteroid is an extended object, typically the brightest in the sky and the pixels are most of the time saturated. In general, the center of mass (CoM) does not coincide with the center if brightness (CoB) of the image and the same irregular shape and rotation of the NEO along with the phase angle causes an offset between the CoM and CoB that can be significant for non-convex asteroids, in the order of the asteroid characteristic radius (see lower right graph of Fig.1). If an altimeter is mounted, during the close approach it may provide ranging measurements from a certain distance until insertion. In the latest part of the close approach, right before insertion into orbit, the very low relative velocity makes the perturbing accelerations from the solar radiation pressure (SRP) and the asteroid gravity (much lower for small objects) to have a significant impact in the trajectory dynamics that must be accounted for.
II. Rendezvous Strategy Definition

A. Far approach

This sub-phase is characterized by a punctual asteroid, very faint at the beginning and therefore with high risk of losing it in the auto-tracking due to the variability of the brightness. In addition, the angles-only navigation provides poor observability of some components of the state vector and poses difficulties in the achievement of the close approach starting conditions with the necessary level of accuracy.

To cope with the asteroid detectability problem, the first mode is Target Detection and Identification (TDI) where the SC is maintained in inertial 3-axis stabilized attitude with the boresight of the navigation camera pointed towards the expected position of the asteroid at the beginning of the autonomous rendezvous phase. No maneuvers are executed and the main objective is the IP to confirm the detection of the asteroid. This mode is basically defined by the initial position and velocity and its duration.

The initial distance is limited by the uncertainty in the interplanetary navigation and the asteroids ephemeris. It is required that the asteroid is within the field-of-view (FOV) during the TDI with 99.7% probability. Therefore, the minimum initial distance for a certain FOV is limited by the relative position uncertainty. The maximum distance is limited by the magnitude of the asteroid in the detector. Using the absolute magnitude of the asteroid, the apparent magnitude can be computed considering the phase angle, the heliocentric distance of the asteroid and the relative distance. This magnitude must be above the limit magnitude of the optical camera and also above the sky magnitude in the region of the observation. In summary, the maximum distance is limited to assure the detectability of the asteroid against the background by assuring a minimum contrast with the sky brightness. Results for a rendezvous with asteroid 2003 SM84 are presented in Fig.2, where the left figure shows the relationship between the FOV of the navigation camera, the phase angle and the available initial distance range.

The initial approach velocity is also constrained to avoid that the asteroid moves out of the FOV before it has been identified. Considering an initial distance, the maximum approach velocity must assure that during the time allocated for the TDI phase and considering the abovementioned relative position uncertainty, the asteroid will remain within the boundaries of the FOV (the starting position in the detector plane consider the 3-sigma relative position uncertainty). In addition, it is considered a minimum approach velocity to avoid imparting accelerating maneuvers, i.e. given the total allocated time to the rendezvous phase and the selected initial distance, a minimum approach velocity without braking maneuvers is calculated. The results of the 2003 SM84 scenario assuming an initial distance of 1e6 km are depicted in the right picture of Fig.2.

Finally, the transversal velocity in the detector plane is limited by observability conditions. The minimum transversal velocity is such that produces a minimum pixel motion between two images and the asteroid can be
identified against the stars. The maximum transversal velocity is limited to avoid that during the integration time of an image the asteroid moves out of a pixel (with a certain probability), so that all the photons from the asteroid are concentrated in the same pixel. The left picture of Fig.2 shows the transversal velocity boundaries and also the pixel motion for the scenario transversal velocity (40 m/s).

The angles-only navigation, i.e. the only measurements are the LOS between the asteroid and the SC (the offset between the asteroid CoB and CoM is negligible in this phase since the target angular size is less than few pixels), poses some problems that must be tackled in the early design of the approach strategy. To achieve full state observability several strategies are possible,

1) Infer distance from target brightness, this measurements have a high error due to the poor knowledge of the asteroid brightness curve in particular its dependence on other factors such as phase angle,
2) Solve an orbit determination problem analogous to the Laplace problem, the main disadvantage is that a significant portion of the asteroid orbit is required for good estimation of the relative trajectory and during this time no maneuvers are executed for accurate estimation,
3) Use a kinematic filter that estimates the LOS and its derivatives and processes them together with known impulsive maneuvers for full relative state determination. It is not applicable to the far approach because the Sun gravity gradient has a significant impact in the relative trajectory (typical duration is several weeks),
4) Follow a dogleg trajectory so that the relative geometry changes allowing full relative state estimation.

The latest option is preferred for autonomous rendezvous because is more generic (valid for impulsive and low-thrust options), does not require a long initial uncontrolled phase and does not need accurate a priori asteroid information. The proper design of the approach trajectory that gives good observability of the relative trajectory must fulfill several conditions,

1) The trajectory is never in course of collision, i.e. the LOS is never aligned with the relative velocity,
2) The required maneuvers to follow the trajectory are decreasing in size with the relative distance and do not introduce large perturbations in the velocity estimation,
3) The phase angle is reducing to start the close approach as close as possible to ‘full disk’,
4) The dogleg maneuvers are such that no large penalties are incurred, compared to the minimum defined by the arrival relative velocity,
5) Enough time to perform navigation is allocated between maneuvers, very important in the case of low-thrust propulsion because of the time to deliver the required delta-V.

An example of such approach for a mission to the 2003 SM84 asteroid is presented in Fig.3, the left picture depicts the trajectory in the asteroid orbital plane in rotating frame, the X-axis points from the asteroid to the Sun.
and the phase angle is better seen, and the right picture depicts the same trajectory in the inertial frame, where the
dogleg maneuvers are clearly appreciated.

![Relative position of the SC wrt the Target](image)

Figure 3. Relative trajectory in asteroid orbital plane in rotating frame (left) and inertial frame (right) for a

B. Close approach

In contrast with the far approach, during the close approach the asteroid is an extended object in the detector
plane and the CoB-CoM offset is the main source of LOS-navigation error. In this mode there is no problem of
observability, on the contrary the pixels of the asteroid are typically saturated, even with much lower exposure
times, and there are no visible stars in the background. To illustrate the problem of the LOS offset, Fig.4 shows two
different frames with different asteroid rotational states. The position of the CoB with respect to the CoM varies
from nearly one radius to zero. In addition, it is very important for the far approach the change in the brightness
induced by this motion, because when the asteroid is in a state like in the left picture the risk of losing the target is
very high (the target is effectively undetectable).

The same observability problem of the far approach is applicable here but if an altimeter is mounted, from a
certain point it would provide direct distance observations that would improve the navigation solution. Nevertheless,
considering that most of the close approach will be carried out with camera-only navigation, the trajectory is
designed to provide the best delivery performances with angles-only navigation. In this way, the approach strategy is
robust against altimeter failures or if the altimeter is not finally mounted. The same conditions for a dogleg
trajectory giving good GNC performances described in the far approach are applicable here.

It is important to note that in this mode, the velocity is slow (from about 10 m/s to 0.1 m/s) and the perturbations
become significant, above all the SRP. This fact should be considered for the achievement of a safe orbit insertion
of about several radius altitude. In this autonomous strategy hold-points are not inserted in this phase, although
given the low relative velocity the cost might be affordable. It is preferred to perform the approach in a fast way
without accelerating maneuvers (in the case of electric propulsion it might require a significant extension of the
phase duration).

Finally, given the usually high uncertainty in the asteroid parameters, in particular the mass that is fundamental
for safe orbit insertion, there is the possibility of performing a low-velocity, close swing-by before the final
insertion. In this case, it is recommended good geometry design for maximizing the accuracy of the velocity change
with radiometric measurements from the Earth, typically Doppler shift and that the incoming and outgoing
hyperbola branches have a phase angle not far from 90º so that the camera observations are not degraded during the
first approach and in the subsequent post-swing-by approach.
III. Autonomous GNC Algorithms

The on-board algorithms include Image Processing to obtain the LOS measurements, navigation that process the LOS and the range if it is available to estimate the relative state and other useful parameters, guidance that compute the maneuvers to cancel the deviations at the desired points and the control that monitors the execution of the maneuvers. These algorithms are described next for the different modes of operation.

A. Far Approach

1. Image Processing

During the TDI mode the attitude command is inertial 3-axis stabilized with constant pointing of the navigation camera boresight in the expected direction towards the asteroid. The time between images is kept constant to facilitate the IP algorithm function.

In order to have sufficient signal-to-noise (SNR) ratio, a stacking of short-exposure frames is preferred over long integration time because of the less tight requirements on the attitude control system (ACS) and to avoid blooming (caused by very bright stars). With the stacking technique the SNR increased with the square root of the number of stacked frames.

The sequence of operations of the stacking process is initiated by the calibration of the raw image by subtracting a bias frame (computed at the beginning), then the calibrated image is co-aligned with a common reference attitude, and finally the co-aligned calibrated image is added to the stack. The main constraint for this stacking process to be effective is to avoid motion of the pixels during the stacking, which translates into a maximum transversal velocity (mentioned in previous section) and the jitter during the exposure time. The effect of the stacking is clearly visible in Fig.5 where the final stacked image shows clearly many stars and also the faint asteroid (green square marker), while the raw image shows mainly the bias and noise.

![Figure 4. Asteroid frames in still geometry save for the rotational state of the asteroid. Compare with Fig.1](image)

![Figure 5. Left: raw image with short exposure (0.95 s) at beginning of TDI mode. Right: result at the end of the stacking process](image)
For the detection of the asteroid against the starry, noisy background a disparity analysis technique is implemented. Initially a difference image of two co-aligned stacks is computed, in this image there are positive and negative peaks corresponding to the brightest pixels in the new and old stacks, respectively. A rim around the difference image is removed to avoid the appearance of many false peaks due to the jitter. Then a thresholding on the positive and negative peaks is carried out separately. The threshold on the photo-electrons measured by individual pixels is the mean plus \( N \) times the standard deviation, with an upper bound that is a fraction of the maximum count. The difference image and the effect of the thresholding are shown in Fig.6.

From Fig.6 follows that several positive and negative peaks appears after thresholding the difference image. Therefore, the asteroid cannot be identified and a candidate selection must be carried out. The filter is based on a minimum and maximum distance between the negative and positive peaks, defined based on the jitter and transversal velocity motion. The peaks passing this filter are stored, note that for each positive peak all the negative peaks that fulfills the distance filter are stored.

For identification of the target among the candidates an analysis of the candidates from the two latest difference images is carried out. A sequence of filters reduce the set of possible candidates, after each filter the number of candidates in the feasible set is checked to stop if there is only one or if it is empty. In the numbering of the feasible candidates a blobbing considering the jitter reduces the number of candidates and avoid spurious multiplicity or ambiguity in the identification. The first filter checks the maximum distance between current and previous candidates, determined by the estimated transversal velocity (in the detector plane). The remaining candidates from the previous and current disparity analysis after the first filter are depicted in Fig.7 showing the several candidates in the TDI mode due to the high level of noise and bright stars. The next filter checks the minimum distance between the negative peak of the current candidate and the positive peak of the previous candidate for persistency of the candidates in the images. The last filter checks the linear motion of the candidates in the two difference images.
The LOS measurement is computed by a centroiding of the pixels in a search box in the difference image that gives the CoB in the detector plane. The star-tracker (STR) estimated quaternion is used to rotate the LOS from detector frame to inertial frame.

It is important to note that 3 stacks are required for initial identification of the target, at the beginning of the TDI mode or in case of restart due to loss of the target. The proposed target is based on kinematic relations and is configurable by a reduced set of parameters, avoiding complex casuistic depending on scenario characteristics. It is generic in the sense that avoids particularizations and robust against false detection or tracking since each disparity produces its own set of candidates independently of the result of the previous disparity analysis or identification process.

The IP algorithm for TDI mode needs some tailoring for the closed-loop far approach mode. The main modifications of the IP algorithm described above are the inclusion of an auto-tracking function to follow the asteroid with the camera and modifications in the identification function to adapt to the changing geometry.

The auto-tracking algorithm uses the relative state from the navigation filter to point the navigation camera towards the estimated position of the asteroid. In this case the rotation between the previous stack and the current one can be significant, which translates in a narrower effective FOV, and the time between stacks must be controlled to avoid the lack of overlapping background in both stacks which will invalidate the disparity analysis.

The identification algorithm is tailored to adapt to the decreasing relative distance that makes the asteroid brighter but the pixel motion increases hyperbolically. During the far approach the star background changes continuously and very bright stars can get in the FOV creating detectability problems as is seen in the left plot of Fig.8, which depicts the electron counts statistics together with the asteroid pixel counts. It shows the difficulty of a far detection because the presence of bright stars and the jitter makes the asteroid very close to the noise and the appearance of brighter stars changes the counts distribution suddenly. The stack size adapts by configuration to the decreasing distance, reducing the time to produce an output and to waste the on-board resources. The effect of reducing the stack size is clearly appreciated in the median.

Finally, the effect of the maneuvers must be considered in the configuration of the parameters of the detection and identification filters because the pixel motion is strongly disturbed due to the sudden decrease of the impact parameter as is seen in the right plot of Fig.8. Apart from the configuration parameters of the image processing (distance thresholds, time between stacks), the auto-tracking algorithm is reset after each maneuver to avoid detectability problems due to the lack of overlapping background.

![Figure 8. Far approach of 2003 SM84 scenario. Left: statistics of the distribution of photo-electron counts and asteroid counts during (from closed-loop simulation). Right: pixel motion per day during nominal trajectory](image)

2. Navigation

The navigation objective is to estimate the full relative state and an unscented Kalman filter (UKF) has been selected as the most suitable option for this phase. UKF has the same characteristics of the extended Kalman filter but with the advantages of not requiring partials computation, therefore the implementation is more generic and easily re-usable or extendable, naturally including non-linear measurements (such as LOS and range) and approximates the statistics to medium order, hence not requiring iterations for improving the performances.
The computational load of the time update (propagation) in the UKF is equivalent to the EKF because a generalized unscented transformation has been implemented, which includes the mean along with the $2N$ sigma points for the covariances construction. Therefore, the number of propagations is $(2N+1)$, the same than in a EKF computing the transition matrix via central differences. The main difference in terms of computational load is the Cholesky factorization required for the sigma-points computation, but it is a negligible term for the typical dimension of the augmented state vector.

The augmented state vector in the far approach sub-phase has 9 components, the SC asteroid-relative state (6-dimensional) and an acceleration bias (3-dimensional), which accounts for unmodeled non-gravitational accelerations and might be significant for electric propulsion model errors.

The time update of the state vector is carried out integrating the equation of motion with a fixed-step, 4th-order Runge-Kutta numerical scheme. The dynamics of the SC state vector includes the Sun and asteroid central gravity, the maneuvers (impulsive or low-thrust) and the acceleration bias. The covariance includes the maneuver execution errors and an additive model noise in the velocity.

The measurement update starts with the computation of the sigma points with the a priori covariance and then the measurements covariance is computed. An additive process noise is included in the measurement covariance to account for the IP error in the determination of the asteroid LOS. Before the update via Kalman gain, a measurement check is passed to reject wrong measurements. This filter is configured carefully to avoid the navigation to stuck in a star instead of following the asteroid.

3. Guidance & Control

The guidance algorithm used with chemical propulsion assumes impulsive maneuvers and can be defined as waypoint-based, differential corrective guidance. The approach is defined by set of waypoints with their associated times, which are pre-defined after mission analysis for each specific scenario to fulfill the requirements of relative state observability (navigation) and asteroid detectability (IP). The corrective maneuvers are applied at specifed control points in order to reach next waypoint at desired time. Note that the velocity control is implicit in the formulation of the guidance problem since it is controlling simultaneously the position and the time of pass.

At a control point the maneuver that cancels the position deviation at the next waypoint is computed as a differential corrective impulse. The dynamics included in the state prediction at the corresponding waypoint includes the Sun 3rd-body acceleration (gravity gradient) and a uniform acceleration that is the acceleration bias estimated by the navigation algorithm.

In the case of electric propulsion, the guidance algorithm is defined as waypoint-based, optimal predictive guidance. Again the approach profile is defined by a set of waypoints and corresponding times, designed during the mission analysis to satisfy the low-thrust specific constraints and performances of the IP and navigation functions. At a control point the optimal thrust profile is computed to reach the next waypoint at the desired time. The thrust direction and the acceleration profile optimize the propellant consumption. The acceleration profile assumes two level of thrust, the highest at the beginning and the lowest at the end (the later can be zero). The dynamics for the propagation of the relative state vector includes the Sun 3rd-body gravity gradient and a constant acceleration accounting for non-gravitational forces.

In the far approach, the maneuvers computed by the guidance are always executed by the main propulsion system, in both cases of chemical and electric propulsion. The main thruster is not aligned with the camera boresight and the SC rotation to achieve the required orientation introduces a delay in the beginning of the execution.

The control of the execution of the impulsive maneuvers is based on the accumulated delta-V measured by the accelerometers. There is a limit in the minimum delta-V that can be delivered in a control cycle due to the accelerometer precision and the RCS residual errors.

The control of the maneuvers with the electric propulsion is time-based because the velocity variation is very low for standard accelerometers. Therefore, the number of control cycles is computed based on the nominal delivered thrust and the estimated mass.

B. Close Approach

1. Image Processing

The detection & identification is much simpler in this mode since the asteroid is the brightest object in the sky. As a result the exposure time is shorter (0.1 s) and there is no need of stacking (in fact in the last part of the far approach the stack size is the one image). In addition, the thresholding does not have to consider the loss of the target in the noise background and a fraction of the distribution close to the maximum value is considered. The value of the threshold must be selected so that the faintest pixels of the asteroid (e.g. near the terminator) are not removed when the asteroid fills a significant portion of the FOV.
The auto-tracking is basically the same than in the far approach based on the estimated relative position from the navigation algorithm. Hence, the navigation accuracy must be such that the asteroid is completely within the camera FOV in the next frame. It is important to note that the time between images is shorter when the SC is closer to the asteroid.

The LOS computation is again a centroiding on adaptive search-box to remove the possible bright stars in the FOV that will bias the CoB position. The search-box size considers the navigation error and attitude pointing error & stability. The search box is depicted in red in the pictures of Fig.9 that shows the images immediately before the centroiding that will give the CoB at the beginning of the close approach and near the end (before orbit insertion).

2. Navigation

The unscented Kalman filter of the far approach is tailored to the close approach, being the main difference that the LOS measurements point to the CoB and not to the CoM (in the far approach this difference is unobservable) and that there is the possibility of having altimeter measurements at different rate than the camera LOS. Hence, the augmented state vector comprises the SC relative state, an acceleration bias, the CoM-CoB offset modeled as an exponentially correlated random variable (ECRV), and the altimeter offset modeled also as an ECRV. The measurement update is sequential and can consider LOS-only, range-only or LOS+range measurements.

In the impulsive close approach a simple formulation of the rendezvous problem can be derived. Based on the assumption of uniform motion, good hypothesis for the short duration of the close approach, we can derive the kinematic relations of Eqs.1 between the variables that define the rendezvous problem (depicted in Fig.10).

\[
\begin{align*}
\tan \theta &= \frac{2\dot{\theta}}{\ddot{\theta}} \\
v/b &= \theta / \sin^2 \theta \\
t_{GO} &= \sqrt{v/b} \tan \theta \\
\vec{v}/v &= \cos \theta \cdot \vec{L}_{OS} - \sin \theta \frac{\vec{L}_{OS}}{||\vec{L}_{OS}||}
\end{align*}
\]  

Figure 10. Kinematic rendezvous problem

The derivatives \( \dot{\theta} \) and \( \ddot{\theta} \) are the norm of the LOS-rate and LOS-rate-rate, respectively. For the estimation of the required LOS derivatives from the LOS readings a low-pass filter is implemented that estimates the LOS by a convolution of the sequence of LOS measurements with the appropriate low pass filter kernels\(^3\). The selected
convolution kernel is the sinc function and the estimated derivatives of the LOS are obtained by convolution of the LOS measurements with the derivatives of the sinc kernel. The estimated values of the LOS and its derivatives must fulfill some constraints.

1) LOS is a unit vector,
2) LOS and LOS-rate are orthogonal,
3) LOS-rate-rate is perpendicular to the LOS and included in the plane of motion

For the complete estimation of the relative state, only the speed or the impact parameter needs to be estimated. The estimation of the speed can be performed based on the execution of known impulsive maneuvers. There are two possibilities for the estimation of the speed after the maneuver, always using the estimated velocity direction before the maneuver.

1) Use the a priori information of speed before the delta-V for initially estimating the speed after the delta-V.
2) Use the estimated after-maneuver velocity direction from the LOS filtering to compute the after-maneuver speed.
3) Weighted contribution of both after-maneuver speeds for the estimated after-maneuver speed

These methods lead to different results and the weighted method has been implemented because is more flexible and by weighting factor configuration can be any of the other two.

Then, the LOS filtering providing LOS-rate and LOS-rate-rate together with known delta-V execution gives the full estimated state. The configuration parameters of the filter are the cut-off frequency of the low-pass filter of LOS derivatives and the weight of the a priori speed estimation.

Before the evaluation of the kinematic filter in the high-fidelity closed-loop simulator, an analysis of the performances in a benchmark scenario (asteroid 2003 SM84) was conducted. The results of the simulations show that the kinematic filter requires significant trajectory deflections in order to have a good estimation of the impact parameter, leading to a kind of spiraling approach as is shown in Fig.11. In addition, the filter requires large quantity of LOS readings and is highly sensible to the CoB-CoM offset. In summary, the kinematic filter gives poorer performances than the UKF, in particular in the final part close to the orbit insertion, and is less flexible because is highly sensible to trajectory changes and is hardly applicable for low-thrust scenarios. For all these reasons the kinematic filter was not investigated in more detail and the effort was focused in the UKF.

3. Guidance & Control

The same algorithms of the far approach are used in the close approach but the impulsive maneuvers are executed with the reaction control system (RCS). The small ∆Vs in this phase (~1 m/s) can be executed with the small thrust delivered by the RCS with negligible penalization for the maneuver duration and allows a better control of the maneuvers above all in the last ones that are very small (~0.1 m/s).

The control of orbit insertion maneuver is event-based instead of time-based because the insertion errors are decreased. The insertion point is defined as the last waypoint in the synodic frame (rotating frame with X-axis always pointing in the Sun direction). The previous waypoint target the insertion point at the desired time but the insertion maneuver is executed at B-plane detection, i.e. when the LOS is perpendicular to the estimated velocity. The necessary delta-V to achieve the desired orbit is computed considering the estimated state and the assumed gravity parameter of the asteroid.
IV. Simulation Capabilities and Scenarios Configuration

A. High-Fidelity Simulator (CHILON††)

The performances of the GNC system are validated in a high fidelity simulator upgraded from a previous tool for impact missions. The hybrid (continuous/discrete), multi-rate simulator has the GNC software (SW) included as Embedded Matlab Functions®, allowing efficient Monte Carlo simulation and also debugging of a particular case. For instance, the execution of a Monte Carlo shot of the far approach (150 days) with the compiled simulator takes about 20 CPU minutes in the chemical propulsion scenario or about 1 CPU hour in the electric propulsion scenario.

The STR model considers the stars from the Tycho-2 catalogue, after inclusion of the orbital aberration, which locate within the FOV and are brighter than the limit magnitude of the sensor. The stars directions are affected by the detector geometrical distortion and a Noise-Equivalent-Angle error. The quaternion defining the attitude of the SC with respect to the inertial frame of the stars map is estimated with the q-method.

The navigation camera model generates the input frames to the IP algorithm and the implementation has been optimized for faster execution time. Initially the camera generates a perfect still image of the asteroid considering realistic illumination conditions, including the self-shadowing (significant for non-convex shapes), and adds the real star background (Tycho-2 catalogue) considering the occultation of stars by the asteroid. Then, the camera measurement effects are added to the perfect image,

1) Optics distortion (Fig.12) and transmittance factor
2) Motion blur during the integration time
3) Gaussian blur in case of defocused optics (i.e. if the STR is used as navigation camera)
4) CCD effects (conversion of photons to electrons): bias, gain inhomogeneity, dark current, stray-light, quantum efficiency, fill factor, miscellaneous additive noise
5) Electronics effects (electron count): saturation, read-out noise, quantization (digitization)

Figure 12. Some optics distortion terms, exaggerated for visualization

B. Benchmark Scenarios

For validation of the GNC performances, two scenarios are configured, one targeting the asteroid 99942 Apophis using electric propulsion and the other one targeting 2003 SM84 with chemical propulsion. Table 1 shows the main characteristics of both scenarios, the auto-rotation period of asteroid 2003 SM84 is not available and a very fast rotation is assumed to have more challenging scenario (note that this scenario is much more demanding for the GNC than the Apophis one because of the physical parameters of the asteroid).

The characteristics of the navigation camera are configured so that the heritage from other ESA missions can be re-used and no specific technology developments are required. In addition, a small and light camera is desired and the parameters affecting size and/or weight are chosen accordingly. A selection of the most relevant characteristics is presented in Table 2, along with the characteristics of the navigation cameras of other ESA missions.

†† GMV/ESA co-funded project developed under contract Nº 1946 (CCN1).
Table 1. Main characteristics of the benchmark scenarios for GNC performances validation

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Apophis</th>
<th>2003 SM84</th>
</tr>
</thead>
<tbody>
<tr>
<td>Propulsion system</td>
<td>Electric: 200 mN</td>
<td>Chemical: 40 N (far approach)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10 N (close approach)</td>
</tr>
<tr>
<td>Pericenter radius of NEO (AU)</td>
<td>0.746</td>
<td>1.033</td>
</tr>
<tr>
<td>NEO characteristic radius (m)</td>
<td>135</td>
<td>57.5</td>
</tr>
<tr>
<td>Albedo</td>
<td>0.33</td>
<td>0.33</td>
</tr>
<tr>
<td>NEO rotation period (hour)</td>
<td>30.53</td>
<td>2.2 (guess)</td>
</tr>
<tr>
<td>Heliocentric distance at arrival (AU)</td>
<td>0.773</td>
<td>1.065</td>
</tr>
<tr>
<td>Initial position for far approach</td>
<td>Distance: 1.02e6 km</td>
<td>Distance: 0.8e6 km</td>
</tr>
<tr>
<td></td>
<td>Phase: -78.5º</td>
<td>Phase: 80º</td>
</tr>
<tr>
<td>Initial velocity for far approach</td>
<td>Radial: 387 m/s</td>
<td>Radial: 477 m/s</td>
</tr>
<tr>
<td></td>
<td>Transversal: -30 m/s</td>
<td>Transversal: 40 m/s</td>
</tr>
</tbody>
</table>

Table 2. Main characteristics of the navigation camera in CHILON and some ESA missions

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>CHILON</th>
<th>AMIE (SMART-1)</th>
<th>ROSETTA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aperture diameter</td>
<td>5 cm</td>
<td>1.6 cm</td>
<td>7 cm</td>
</tr>
<tr>
<td>FOV</td>
<td>Far approach: 2º</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Close approach: 13º (Apophis)</td>
<td>2.15º</td>
<td>2.5º</td>
</tr>
<tr>
<td></td>
<td>7º (2003 SM84)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Detector array (pixels)</td>
<td>512x512</td>
<td>512x512</td>
<td>1024x1024</td>
</tr>
<tr>
<td>Readout noise per pixel</td>
<td>30 e−</td>
<td>30 e−</td>
<td>30 e−</td>
</tr>
<tr>
<td>Quantum efficiency</td>
<td>30%</td>
<td>18%</td>
<td>33%</td>
</tr>
<tr>
<td>Dark current</td>
<td>76.462 e−/s</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Exposure time</td>
<td>Far approach: 0.95 s</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Close approach: 0.1 s</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

V. GNC Systems Performance Assessment

A. Target Detection & Identification

1. Apophis

The initial conditions for the TDI mode result from the minimum asteroid brightness detectable by the navigation camera, and are more stringent than in the far approach. The objective is to start the TDI mode as soon as possible and have enough time to assure the identification of the asteroid to start the far approach. The initial conditions corresponding to the configure camera and the Apophis asteroid is 1.4e6 km and 90º-phase for the position, and 380 m/s (radial) and 30 m/s (transversal) for the velocity. The duration of the TDI mode in this simulation is 1 day, the time between stacks is 10800 s and the number of frames per stack is 1024.

A batch of 30 simulations is run and a summary of the results is presented next. The histogram of the number of confirmations is presented in the left picture of Fig.13 showing that in one case the number of successful identifications is not the maximum (6) but only 4. Apart from the number of confirmations it is important to check the correctness of such identifications, the right picture of Fig.13 shows the LOS error between the identified target and the real asteroid position. All the identifications with LOS error greater than 4 pixels (typical size of the blob due to the jitter) are considered wrong. With this criteria the rate of success of the IP algorithm in this scenario is 96.7% (174/180).
2. 2003 SM84

Again the initial conditions are defined by the limit of detectability of the sensor and are not the same than the far approach. In this scenario this initial conditions are 1e6 km and 80º-phase for position, and 477 m/s (radial), 40 m/s (transversal) for velocity. The same TDI duration (1 day) and IP configuration than in the Apophis scenario is set.

A batch of 20 simulations is run. The histogram of the number of confirmations and the LOS error are presented in Fig.14, showing worse results than in the Apophis case. With the same criteria of the previous Apophis scenario, 4 pixels of maximum error, the rate of success of the IP algorithm in this scenario is 71.7% (86/120). This is due to the harder conditions of the 2003 SM84 scenario, mainly the higher variability of the asteroid magnitude because of the faster rotation period, and the fainter asteroid because the higher transversal velocity at closer distance makes much higher pixel motion during the stacking time and the photons are spread over more than one pixel.

3. Conclusions

The previous results are quite encouraging given the in harsh conditions selected to test the TDI mode. In the short duration of the TDI mode all the runs gave correct identification of the target and it is important to note that for good performances of the IP algorithm there is no need of 100% success ratio but only at the end of the TDI give the right LOS towards the target to start the far approach mode being able to track the target.
There are some guidelines that can be derived from the analysis of the results to assure successful target
detection and identification before starting the far approach,
1) Extend the total duration of the TDI phase
2) Start at closer distance
3) Have accurate brightness curves for better configuration of the IP algorithm
4) Increase the stack size and reduce the transversal motion, which will imply an increase in the time between
stacks
5) Use a binning technique to avoid photon dispersion in several pixels
6) Decrease threshold of disparity to reduce the risk of loss of the target but will increase the risk of false
detections and the computational time
7) Use the information of the previous candidates and detected pixels to predict the position in a future frame,
it will improve the recovery after no-detection (target loss) but will be less robust against false detection.

B. Far Approach

1. Apophis

The nominal initial conditions are presented in Table 1 and the dispersion in these initial conditions is 1000 km
in position and 1 m/s in velocity (1-sigma), independent in each direction. In the Apophis scenario the final
conditions for extended asteroid corresponds to a relative position ~2200 km and relative velocity ~10 m/s.

A batch of 10 simulations is executed and the navigation and guidance performances using the configured
optical sensors at the interface with the close approach are summarized below.

1) Position estimation: actual error < 10 km & estimated error ~50 km (1-sigma)
2) Velocity estimation: actual error < 0.02 m/s & estimated error ~0.2 m/s (1-sigma)
3) Deviation at last waypoint (interface): ~20 km (< 1% of distance)

The navigation position error (actual and estimated) in the first run of the batch is presented in Fig.15. The
estimated error always covers the actual error in this simulation. It is important to note the impact of the low-thrust
maneuvers on the navigation, long arcs with no measurements and increase in the covariance. The right picture
shows that after the velocity estimation has reached a good accuracy the position estimation improves steadily when
the distance decreases in all the directions (due to the dogleg maneuvers).

Some guidance performances are presented in Fig.16 for the entire batch of simulations. The deviations at the
waypoints are depicted in the left picture, which are driven by navigation and execution errors at previous waypoint.
The right picture shows the cumulated delta-V and shows that total delta-V budget requires an additional ~30% of
arrival velocity due to the dogleg maneuvers, the gravity losses (non-impulsive maneuvers), and the navigation
ersors.

![Figure 15. Navigation performances of first simulation in the closed-loop far approach mode for Apophis
scenario. Red lines shows the bounds of the estimated error (1-sigma) and blue lines show the actual error.
Left: entire 48-day far approach. Right: zoom of the last 10 days](image-url)
Again the nominal initial conditions are presented in Table 1 and the dispersion in these initial conditions is 1000 km in position and 1 m/s in velocity (1-sigma), independent in each direction. In the 2003 SM84 scenario the final conditions for extended asteroid corresponds to a relative position ~1200 km and relative velocity ~8 m/s.

A batch of 10 simulations is executed and the navigation and guidance performances using the configured optical sensors at the interface with the close approach are summarized below.

1) Position estimation: actual error < 10 km & estimated error ~25 km (1-sigma)
2) Velocity estimation: actual error < 0.01 m/s & estimated error ~0.1 m/s (1-sigma)
3) Deviation at last waypoint (interface): ~100 km (< 10% of distance)
4) Delta-V budget needs an additional ~20% of arrival velocity

The navigation position error is presented in Fig.17, the actual error in the left picture and the estimated error (square root of the corresponding term of the diagonal of the covariance matrix) in the right picture. There are some cases in which the estimated error does not cover the actual error during short periods, which is not strange since the estimated error is 1-sigma. The most important point is that during these periods there are no measurements available (measurement update is indicated by a marker in the right picture of estimated error). In the left plot of Fig.18 the actual LOS error (blue) and the estimated LOS error immediately before the measurement update (red) is depicted. Again the markers indicates the measurement update, the lack of markers when there is no long maneuver are due to the IP not able to identify the target or the navigation algorithm rejecting a measurement because the difference with the expected LOS is higher than the selected threshold. These periods without measurement updates at the beginning (between 5 and 20 days) cause the degradation of the navigation solution.

The guidance performances are presented in the right plot of Fig.18 for one simulation of the batch, the trajectory in different planes is presented and the waypoints highlighted with red markers. The deviations at the last waypoint are higher than in the Apophis case because the configured propulsion system errors are much higher in this scenario, the thrust magnitude error is 1% (1σ) bias + 1% (1σ) ECRV + 1% (1σ) white noise. In addition, the accelerometer measurement, used to control the execution and to include the maneuver in the navigation algorithm introduces and additional 0.01 m/s error that is directly translated into the delta-V cut-off error.

Therefore, with the configured errors, the maneuver execution error at previous control point determines the deviation at the next waypoint. Additional control closer to the final waypoint can be inserted to reduce the final deviation or more precise thruster can be modeled. For the present scenario the performances are good enough to carry out the mission and no further refinement of the GNC system was considered necessary.
3. Conclusions

From the analysis of the Monte Carlo simulations follows that the GNC performances are good enough to begin the close approach phase, since there is never risk of losing the target out of the FOV, nor colliding with the asteroid. The navigation accuracy is sufficient to start the close approach since it is better than 1% of the relative state. Nevertheless, it is important to highlight that there is margin for improving the performances in this phase should it be required, since the initial conditions are very far away and the closed-loop far approach can start closer to the target (considering the mission analysis parameters described in the 2nd section).

The navigation shows good behavior since all the position components are estimated with accuracy equal to the LOS error. There is only navigation divergence in case of total loss of the target (Fig.8 left), which can be prevented by better a priori knowledge which would allow starting the far approach closer to the target. The tracking of false target is handled by the measurement rejection check within the navigation algorithm and does not degrade the performances. It is important to remark that there is no artificial noise in the navigation filter to avoid divergence.

The guidance deviations are low enough to safely start the close approach, since at the last waypoint are always lower than 10% distance even with the configuration of very high execution errors. Some further refinement in the approach strategy and the propulsion system characteristics will give better performances if required. The delta-V budget is the arrival velocity plus a 20% in the case of chemical propulsion and 30% in the case of electric propulsion. The main causes for this additional propellant expenditure in the present simulations are the navigation error at maneuver execution and the dogleg maneuvers for observability of the entire state vector.
C. Close Approach

1. Apophis

Initially the performance analysis of the LOS-only navigation, i.e. without altimeter, is carried out. The initial conditions are taken from the far approach simulations and the final conditions to define the last waypoint results from the requirement to insert the SC in a circular, dawn-dusk orbit of 1000-m radius. A batch of 10 simulations is run and the results are summarized below.

1) Position estimation: actual error < 100 m (Fig.19 left) & estimated error ~200 m (1-sigma)
2) Velocity estimation: actual error < 0.01 m/s (Fig.19 right) & estimated error ~0.02 m/s (1-sigma)
3) Delivery errors <400 m but always inserted into orbit and distance to the target never lower than 800 m (Fig.20 left)
4) Delta-V budget ~13 m/s, an additional 40% of initial velocity

The navigation accuracy improves steadily while the distance is decreasing until the very end when there are other effects associated to the low velocity that makes the situation more delicate. The small maneuvers control and the acceleration bias become crucial (Fig.21 proves the observability of the SRP in the last days of the simulation), so does the trajectory design of the last waypoints.

![Figure 19. LOS-only navigation GNC performances: actual error in position and velocity, in the last 3 days](image1)

![Figure 20. LOS-only navigation GNC performances: delivery errors and waypoint deviations](image2)

Waypoint errors

![Waypoint errors](image3)
Then, the GNC performances using camera and altimeter are analyzed, the altimeter provides range measurements from 20 km. The same initial conditions and dispersion than in the previous LOS-only case are configured and the results are summarized below.

1) Position estimation: actual error < 40 m (Fig.22 left) & estimated error ~20 m (1-sigma)
2) Velocity estimation: actual error < 5 mm/s (Fig.22 right) & estimated error ~0.01 m/s (1-sigma)
3) Delivery errors <100 m (Fig.23 left)
4) Delta-V budget ~13 m/s (Fig.23 right)

The final navigation accuracy with altimeter is nearly the same than with LOS-only navigation and it is determined by the physical limit imposed by the CoB-CoM offset, in this scenario the phase angle is small and the worst case offset is a fraction of the radius. In this case the slow rotation of the asteroid (30 hours) can be seen in the position error. The main benefit of the altimeter (with the model implemented) is the achievement of the best performances shortly after the acquisition of range measurements. With this accurate navigation the guidance maneuvers at the control points are more precise and the final delivery error is lower, nearly reaching the limit of the navigation uncertainty.
In this scenario the performance analysis if the LOS-only navigation from a batch of 10 runs is summarized below. Again, the initial conditions are taken from the far approach simulations and the objective is to insert the SC in a 1000-m circular, dawn-dusk orbit.

1) Position estimation
   a. actual error < 20 m in X, Y axes and <40 m in Z-axis (Fig.24)
   b. estimated error ~20 m (1-sigma) in X, Y axes and ~100 m (1-sigma) in Z-axis

2) Velocity estimation: actual error < 5 mm/s (Fig.25 left) & estimated error ~0.02 m/s (1-sigma)

3) Delivery errors <200 m

4) Delta-V budget ~8.5 m/s (Fig.25 right)

In this scenario the asteroid size is smaller and the final navigation accuracy compared with the Apophis scenario is correspondingly lower. It seems that there is still room for improving the Z-component estimation with a refined trajectory design, in order to reach the physical limit of the CoB-CoM offset that is achieved in the X and Y axes. The delivery errors are conditioned by the navigation accuracy in the previous control point and above all by the maneuver execution, that is too high as has been mentioned previously. It is important to note that the delta-V budget has a very small penalty of about 0.5 m/s with respect to the initial arrival velocity, compared with the significant additional delta-V in the case of electric propulsion.
The performances of the GNC system with the same conditions than in the previous scenario, but considering the altimeter in the sensor suite, are summarized below from a batch of 10 runs. The use of altimeter improves the navigation accuracy (actual error and knowledge) of all the components to the best possible value and reduces also the delivery error to a limit compatible with the navigation accuracy.

1) Position estimation: actual error < 20 m (Fig.26 left) & estimated error ~20 m (1-sigma)
2) Velocity estimation: actual error < 5 mm/s (Fig.26 right) & estimated error ~10 mm/s (1-sigma)
3) Delivery errors < 30 m
4) Delta-V budget ~8.5 m/s

The present approach strategy and GNC system can insert the SC into orbit using LOS-only navigation, even if there is some margin for improvement of performances and the final orbit will have significant deviations from the nominal one. The navigation performances at the insertion point are essentially the same with and without altimeter being the main difference the navigation accuracy at the control points previous to the B-plane waypoint, i.e. the navigation accuracy in the previous hours is more important for the delivery errors than the accuracy at the insertion point itself, that is crucial for the orbit insertion maneuver. The navigation accuracy has a physical limit
defined by the CoB-CoM offset that depends on the rotation axis but is brought to the minimum by a proper approach trajectory in the synodic frame.

The delivery error at the insertion point inherits the previous waypoint navigation error and the use of altimeter brings the final error to the minimum. Considering the navigation performances it seems that the best option is to use an altimeter that has several kilometers of operational range as nominal case but design the approach trajectory and GNC system to be able to insert the SC into orbit around the asteroid without the distance measurements (contingency case).

The delta-V budget in the case of chemical propulsion is basically the same than the initial velocity at the beginning of the phase, while in the electric propulsion there a significant penalty that increases the delta-V budget in 40% of the initial velocity.

VI. Conclusions

A generic autonomous GNC system covering the entire rendezvous phase, from TDI to orbit insertion, with a small NEO has been presented. The GNC system is composed of standard sensors and actuators and includes an efficient implementation of the IP and GNC algorithms. The feasibility of the autonomous GNC has been validated via Monte Carlo simulation in a representative environment (high-fidelity closed-loop simulator). Two different scenarios have been analyzed with different characteristics and the analysis of the obtained performances gives some guidelines for the design of the GNC system in specific missions.

The far approach has proved that a robust IP algorithm against wrong tracking is required because the adverse detectability conditions (in the configured scenarios) can easily result in the loss of asteroid. Several techniques have been proposed to assure the target tracking of very faint asteroids. There is a strong impact of the approach design on the navigation and guidance performances, and the presented strategy provides results good enough to start the close approach and has margin to improve the performances at the interface point. The navigation algorithm has proved robust against loss or wrong tracking in moderate long arcs. The guidance deviations are mainly driven by navigation accuracy and the low-thrust propulsion introduces higher delta-V penalties than the impulsive option.

The close approach analysis shows that autonomous orbit insertion without altimeter is feasible, even if the delivery errors are high the margins in the nominal orbit avoid collision with the asteroid. Again the approach profile is crucial to obtain good observability in all the relative state components. An altimeter with moderate range seems the best option for nominal operations given the better performances (delivery error nearly reaches the physical limit of the CoB-CoM offset) and the lower delta-V budget. It is important to note that the proposed IP and navigation algorithms achieve very good performances without complex scene analysis or landmark tracking and with no need of detailed a priori information of the asteroid (only estimated radius and rotation period).

References