ABSTRACT

All realistic methods to establish random acceleration spectral density specifications on a test item in hardmounted configuration (called equipment test level in this paper) are based on predicted (or measured) random acceleration responses at the interface of the item considered as load in the flight mounted configuration (called system or coupled test level in this paper).

A classical approach is to consider a frequency spectrum envelope by enclosing in a log-log plot the interface response data. This very conservative method usually produces overtesting test specifications with a root-mean-square acceleration (grms) value equal to several times the response curve grms value.

NASA proposes on its FEMCI website an improvement to this method by authorizing sharp peaks to be cut off and trying to keep the overall grms of the specification not far above that of the response curve. This is convenient for low mass equipment like electronic boxes with hardmounted structural modes at high frequency (typically >500Hz where random levels are low and usually decreasing by a slope of ~3dB/octave). On that type of equipment, strength qualification is obtained by a quasi-static qualification.

This is not convenient in the case of equipment with hardmounted main structural modes (high effective mass) at a frequency much below 500Hz which will decrease significantly when integrated on the flight mounting structure. In fact, in this case, force limitation around the high effective mass modes (called primary notching) can be necessary not to overttest the equipment. The rules to obtain the bandwidth and the depth of those random notchings are not obvious and often system authorities are reluctant to accept them.

By applying a force limited specification on the conventional acceleration specification

\[
\text{force limited specification} = \frac{\text{interface force envelope}}{\text{interface acceleration envelope}},
\]

NASA proposes in the handbook “Force limited vibration testing” different methods considering a simple model with only one mode for each of the test item and the mounting structure. For different test item modes, effective mass is also introduced in this handbook but no precise methods are detailed.

This paper describes a method based on effective mass superposition to derive a realistic random specification integrating different primary notching requirements on a hardmounted equipment.

This method identifies on the global interface force at system level the contributions of the main structural modes, subtracts them to the global force, “translates” them at equipment level, and finally reintroduces them in the global interface force which becomes an “equipment interface force”. This equipment global interface force will be then transformed into an equipment mean interface acceleration (with the equipment force transfer function) which will be used to create the interface acceleration specification by for instance the NASA FEMCI method.

This method guarantees to reach the same stress repartition on the equipment between the system and the equipment level.

This method is illustrated by the example of a laser source (with a suspended optical bench) integrated in a payload submitted to random environment, inspired from the development of the PHARAO/ACES (Atomic Clock Ensemble in Space), COLOMBUS payload.

1. INTRODUCTION

The first idea of that paper was to find out a method to specify in random a simple equipment (equipment in embedded configuration) from the results of analysis at a system level (equipment mounted on a complex structure).

This is a common question all the mechanical architects of a system ask themselves when they have to specify the environment of the sub-assemblies of their system.
Usually, they use for each axis the average (or sometimes maximum) interface acceleration of the equipment on the system, and define a skyline surrounding it which they consider as the environment specification. Sometimes, they use the NASA method [5], cut the peaks at 3db and try not to exceed 1.25 times the overall RMS acceleration of the interface acceleration.

Anyway, when the equipment is more complex than a single electronic box, has heavy effective mass and low frequency modes, equipment supplier often ask for “primary notchings” in random at these heavy modes frequencies not to over design the equipment.

This paper proposes a method which helps the mechanical architect to define the position and the depth of these notchings.

2. MODAL SUPERPOSITION THEORY

2.1 Principle of modal superposition

Let us consider a linear structure with its interface. This structure is submitted to the equation of motion (1):

\[ M \ddot{q} + C \dot{q} + K q = F \]  

(1)

where \( q \) are the DOFs (degrees of freedom) of the structure, \( M \), \( C \) and \( K \) the mass, damping and stiffness matrices respectively and \( F \) the external loads.

The set of DOFs can be partitioned in two subsets:
- the interface DOFs (j subset)
- the internal DOFs (i subset)

The structure is submitted to base motion (\( u_j \)) and internal forces (\( F_i \)). The dynamic analysis objective is to obtain the responses of internal displacement (\( u_i \)) and the reaction at the interface (\( R_j \)).

\[
\begin{bmatrix}
  u_j \\
  u_i
\end{bmatrix} = 
\begin{bmatrix}
  1 & 0 \\
  \phi_{ij} & \phi_{ik}
\end{bmatrix}
\begin{bmatrix}
  u_j \\
  \eta_k
\end{bmatrix} = 
T_{ij}
\begin{bmatrix}
  u_j \\
  \eta_k
\end{bmatrix}
\]

(2)

with \( \phi_{ij} \) the junction static modes matrix, and \( \phi_{ik} \) the normal mode matrix (at \( u_j \) fixed)

By introducing the normal modes of the structure (2) clamped at its interface, in the frequency domain \( \omega \), the relationships between all these parameters can be written using FRF (Frequency Response Functions) (3):

\[
\begin{bmatrix}
  G_{ii}(\omega) \\
  T_{ij}(\omega) \\
  R_{ij}(\omega)
\end{bmatrix} = 
\begin{bmatrix}
  \tilde{G}_{ii}(\omega) & \tilde{T}_{ij}(\omega) \\
  -\tilde{T}_{ji}(\omega) & -\omega^2 \tilde{M}_{jj}(\omega) + \tilde{K}_{jj}
\end{bmatrix}
\begin{bmatrix}
  F_i(\omega) \\
  u_j(\omega)
\end{bmatrix}
\]

(3)

where \( G_{ii} \), \( T_{ij} \) and \( M_{jj} \) are the dynamic flexibility, transmissibility and mass matrices respectively (\( T_{ij} = T_{ji} \)) and \( K_{jj} \) the reduced stiffness at the junction.

\[
\tilde{G}_{ii}(\omega) = \sum_{k=1}^{\infty} H_k(\omega) \frac{\phi_{ik} \phi_{ik}}{\omega_k^2 m_k} \]  

(4)

with \( \tilde{G}_{ii}, k = \frac{\phi_{ik} \phi_{ik}}{\omega_k^2 m_k} \) effective flexibility of mode \( k \)

\[
\sum_{k=1}^{\infty} \tilde{G}_{ii}, k = G_{ii} = K_{ii}^{-1} \text{ static flexibility matrix}
\]

\[
\tilde{T}_{ij}(\omega) = \sum_{k=1}^{\infty} \frac{\phi_{ik} L_{kj}}{m_k} \]  

(5)

with \( \tilde{T}_{ij}, k = \frac{\phi_{ik} L_{kj}}{m_k} \) effective transmissibility of mode \( k \)

\[
\tilde{M}_{jj}(\omega) = \sum_{k=1}^{\infty} \frac{\phi_{ik} \phi_{ik}}{\omega_k^2 m_k} \]  

(6)

with \( \tilde{M}_{jj}, k = \frac{L_{kj} L_{kj}}{m_k} \) effective mass of mode \( k \)

\[
\sum_{k=1}^{\infty} \tilde{M}_{jj}, k = M_{jj} - (M_{jj} - M_{ji} M_{ij}^{-1} M_{ij}) \text{ static mass – junction mass matrix}
\]

and \( K_{jj} = K_{jj} - K_{ji} K_{ii}^{-1} K_{ij} \) junction stiffness matrix

where \( m_k \) is the generalized mass of mode \( k \), \( H_k \) (8) and \( T_k \) (9) are the amplification and transmissibility factors of mode \( k \), expressed from the pulsation \( \omega \) and the modal damping ratio \( \zeta_k \) (7) and (8) and \( L_{kj} \) is the modal participation factor matrix:

\[
H_k(\omega) = \frac{1}{1 - \left( \frac{\omega}{\omega_k} \right)^2 + i.2\zeta_k \left( \frac{\omega}{\omega_k} \right)}
\]

(8)

\[
T_k(\omega) = \frac{1 + i.2\zeta_k \left( \frac{\omega}{\omega_k} \right)}{1 - \left( \frac{\omega}{\omega_k} \right)^2 + i.2\zeta_k \left( \frac{\omega}{\omega_k} \right)}
\]

(9)

\[
L_{kj}(\omega) = \phi_{ki} (M_{ii} \phi_{ij} + M_{ij})
\]

(10)
The terms outside of the sum in equations (5) and (6) are reduced masses at the junction and often negligible.

When modal analysis is performed, only a short number of eigenfrequencies (m modes) is computed. This is a truncation of the modal base, the contribution of the higher modes can be represented from (4) (5) and (6) by residual terms.

\[
\tilde{G}_{ii,\text{res}} = K_{ii}^{-1} - \sum_{k=1}^{\infty} \tilde{G}_{ii, k}
\]  

(11)

\[
\tilde{T}_{ij,\text{res}} = \phi_{ij} + M_{ii}^{-1} \cdot M_{ij} - \sum_{k=1}^{\infty} \tilde{T}_{ij, k}
\]

(12)

\[
\tilde{M}_{jj,\text{res}} = M_{jj} - (M_{jj} - M_{ji} \cdot M_{ii}^{-1} \cdot M_{ij}) - \sum_{k=1}^{\infty} \tilde{M}_{jj, k}
\]

(13)

### 2.2 Evaluation of the global interface force on the equipment

At equipment level, with software such as “MSC/NASTRAN”, it is easy to evaluate the global interface force: all the interface points of the equipment are linked by rigid elements (RBE2) to a major point which is itself linked by an important stiffness spring to heavy mass to simulate an embedded interface:

![Diagram of equipment with Rigid body elements and Heavy mass connected by null length spring](image)

At system level, the way to get this interface force is not as simple. The most precise method is to evaluate the interface force transfer function at each interface point and to sum all of them to get the global interface force transfer function.

![Diagram of system structure with Equipment (i), Null length spring elements to measure the single interface force transfer function](image)

The best way to perform this summation is to use the junction static modes matrix \( \phi_j \) which allows to evaluate on a point \( i \) the equivalent tensor (force and moment matrix) of the global interface:

In case of static motion of the interface, we get (14) for the motion of an internal point \( i \).

\[
u_i(\omega) = \phi_{ij} \cdot u_j(\omega)
\]

(14)

On the contrary, in case of a static motion of a single point called \( s \), if the structure is considered as rigid, the interface points will get the following motion (15):

\[
u_j(\omega) = \phi_{js} \cdot u_s(\omega)
\]

(15)

Reaction on this single point is calculated by the following equation (16):

\[
R_s(\omega) = \phi_{sj} \cdot R_j(\omega)
\]

(16)

\( \phi_s \) is easy to calculate (17):

\[
\phi_{sj} = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & z_j + z_s & y_j - y_s & 1 & 0 & 0 \\
z_j - z_s & 0 & -y_j + y_s & 0 & 1 & 0 \\
y_j + y_s & x_j - x_s & 0 & 0 & 0 & 1
\end{bmatrix}
\]

(17)

with \((x_s,y_s,z_s)\) the co-ordinates of the “single” point and \((x_j,y_j,z_j)\) the co-ordinates of all the interface points.

This “single” point where the global tensor will be evaluated can be either the average interface point, but is more interesting at the equipment center of mass location (COM) which prevents from including in the global reaction the “moment effect” coming from the COM height from the interface. The COM global forces can directly be compared to the random specifications of the equipment.

This method will be also applied at equipment level.

Nota: in the paper, the force interface function will always be divided by 9.81 and the static mass of the
equipment to be similar to an acceleration transfer function (equals 1. At 0Hz).

2.3 Evaluation of Power Spectral Density transfer function

For random analysis, when \( S_y(\omega) \) is the Power Spectral Density (PSD) of the response, \( S_x(\omega) \) the PSD of the input and \( H_{yx}(\omega) \) the transfer function, we get:

\[
S_y(\omega) = |H_{yx}(\omega)|^2 S_x(\omega)
\]  

(18)

Reaction PSD transfer function

From (3) (6) and (13), in case of solicitation by the interface \((F_i=0)\) we can compute the junction reaction Frequency Response Function (FRF called force transfer function in this paper) on one interface point as follows:

\[
R_j(\omega) = -\omega^2 M_{jj}, \text{res}.u_j'(\omega)
\]

\[
-\omega^2 \sum_{k=1}^{m} T_k(\omega) \tilde{M}_{jj}, k + M_{jj} - M_{ji}.M_{ii}^{-1}.M_{ij}.u_j(\omega)
\]

\[
+ K_{jj}(\omega).u_j(\omega)
\]

(19)

As said before, the reduced masses at the junction are often negligible and when we are interested by the global force at the interface, the sum of the reactions on the interface points coming from the static stiffness \( K_{jj} \) is null. Thus, only the first terms of equation (19) can be considered for the total reaction at the interface:

\[
R_j(\omega) = (\tilde{M}_{jj}, \text{res} + \sum_{k=1}^{m} T_k(\omega) \tilde{M}_{jj}, k).u_j''(\omega)
\]  

(20)

\[
R_j(\omega) = H_{jj}'(\omega).u_j''(\omega)
\]

with the reaction transfer function (21) or (21.1):

\[
H_{jj}'(\omega) = \tilde{M}_{jj}, \text{res}.I + \sum_{k=1}^{m} T_k(\omega) \tilde{M}_{jj}, k
\]  

(21)

or

\[
H_{jj}'(\omega) = \tilde{M}_{jj}.I + \sum_{k=1}^{m} (T_k(\omega) - 1) \tilde{M}_{jj}, k
\]  

(21.1)

Acceleration PSD transfer function

From (3) (5) and (12), in case of solicitation by the interface \((F_i=0)\) we can compute the internal acceleration FRF (called acceleration transfer function in this paper) on one interface point as follows:

\[
u_i''(\omega) = (\tilde{T}_{ij}, \text{res} + \sum_{k=1}^{m} T_k(\omega) \tilde{T}_{ij}, k - M_{ii}^{-1}.M_{ij}).u_j''(\omega)
\]  

(22)

As said before, the reduced masses at the junction are often negligible. Thus, only the first terms of equation (22) can be considered for the internal acceleration:

\[
u_i''(\omega) = (\tilde{T}_{ij}, I + \sum_{k=1}^{m} (T_k(\omega) - 1) \tilde{T}_{ij}, k).u_j''(\omega)
\]  

with the acceleration transfer function (24):

\[
H_{ij}''(\omega) = \tilde{T}_{ij}, I + \sum_{k=1}^{m} (T_k(\omega) - 1) \tilde{T}_{ij}, k
\]  

(24)

3. GENERAL APPROACH

As said in the introduction, the objective of this method is to propose a random specification at equipment level from results of calculations at system level.

- For that, the first step is from the FRFs to identify the different modes characteristics of the equipment, both at equipment level (equipment in hardmounted configuration) and at system level (equipment mounted on a global structure). This can be easily done at equipment level, these parameters being the results of the modal analysis. This task is more difficult at system level, a simple modal analysis will give these parameters w.r.t. the global structure interface acceleration and not towards the average equipment interface acceleration!

- The second step is to correlate the equipment modes frequencies at system level to the equipment modes in hardmounted configuration. This can be done by comparing the global reaction and some internal dedicated points transfer functions (transfer from the equipment average acceleration) and helped by modal deformations.

- The third step is to extract in the global reaction of the equipment at system level the contribution of important effective mass modes which frequency has been shifted between the two configurations. This task is not easy too, as if modal superposition can be easily done on FRF, it is not the case on the PSDs which contains cross terms between the different modes.

- The fourth step is to translate these contributions at equipment level, generate the global reaction we want to obtain at equipment level and by the “force limit method” to get an equivalent PSD acceleration at the equipment interface.

3.1 Identification of modal parameters

The purpose of this paper is not to propose a modal identification method, but anyway a simple method is proposed to illustrate the general approach.
By analysing the FRFs of the global interface force at COM and well located internal points, we can identify for the mode \( p \) at a pulsation \( \omega_p \) defined by operator (especially on the internal points acceleration FRFs) the modal damping ratio \( \zeta_p \) of the modes of the structure. These parameters are easier to identify on internal accelerometer responses than on global interface force because of the effect of the modal superposition on the force which is not so important on accelerations if the internal points are located on separated substructures.

The method used is recursive (2 steps), the \( p \) mode parameters being first identified after subtraction of the \((p-1)\) identified participation and then re-identified after subtraction of the upper modes identified participation (to suppress their low frequency participation). The transfer function which is used to perform this calculation is either (26) for the global interface force or (27) for internal accelerations:

\[
H_{jj}^{+}(\omega) = \sum_{k=p}^{m} (Tk(\omega) - 1)M_{jj}, k \tag{26}
\]

\[
H_{jj}^{+}(\omega) = \sum_{k=p}^{m} (Tk(\omega) - 1)\tilde{T}_{jj}, k \tag{27}
\]

(26) and (27) equations reach a maximum at \( \omega_p \), which is proportional to the maximum of \((Tk(\omega_p)-1)\) (28), the contribution of the upper modes \((p+1 \text{ to } n)\) being insignificant at \( \omega_p \).

\[
Tk(\omega_p) - 1 = \frac{1}{i.2\zeta_p} \tag{28}
\]

The most classical method to identify the modal damping ratio \( \zeta_p \) is the “half power method” (25):

\[
\zeta_p = \frac{(\omega_2 - \omega_1)}{2.\omega_p} \tag{25}
\]

where \( \omega_1 \) and \( \omega_2 \) are the pulsations where \( H(\omega_1) = H(\omega_2) = H(\omega_p)/\sqrt{2} \) with \( \omega_1 < \omega_2 \).

Once \( \zeta_p \) is identified (when possible by accelerometers function (26) otherwise by global force function (27)), we identify at \( \omega_p \) on the global interface force from (26) the effective mass \( M_{jj}, p \), and on internal points accelerations from (27) the effective transmissibility \( T_{ij}, p \) of the mode:

\[
\tilde{M}_{jj}, p = \sum_{k=p}^{m} (Tk(\omega_p) - 1)M_{jj}, k.2.\zeta_p \tag{29}
\]

\[
\tilde{T}_{ij}, p = \sum_{k=p}^{m} (Tk(\omega_p) - 1)\tilde{T}_{jj}, k.2.\zeta_p \tag{30}
\]

This step of identification is necessary to get the modal parameters of the modes we want to shift in frequency. The accuracy on important modes (with large effective mass) is sufficient for the next steps.

### 3.2 Correlation “system” towards “equipment”

For each system mode we want to translate, we define different equipment modes the system mode corresponds to.

![Figure 3](image)

This combination allows to take into account different equipment modes which could have been coupled at system level.

### 3.3 Extraction of modal participation

This step has been first defined at FRF level to be able to perform it at PSD level.

As defined in equation (18), PSD transfer function is the norm of the FRF transfer function (31):

\[
H^{+}_{psd}(\omega) = |H_{jj}^{+}(\omega)|^2 = H_{jj}^{+\ast}(\omega).H_{jj}^{+}(\omega) \tag{31}
\]

From (21), we obtain (32) if we separate the mode \( i \) we want to translate:

\[
H_{ij}^{+}(\omega) = M_{jj}, res.I + \sum_{k=\omega}^{m} Tk(\omega)M_{jj}, k + \tilde{T}(\omega)\tilde{M}_{jj}, i \tag{32}
\]

We will simplify the notation as defined in (33) and omit to write the pulsation:
\[ Mr = \bar{M}_{jj,\text{res}} \]
\[ Mk = \bar{M}_{jj,k} \quad (33) \]
\[ Mi = \bar{M}_{jj,i} \]
(32) becomes (34):
\[ H_{ij}' = Mr.J + \sum_{k \neq i} Tk.Mk + Ti.Mi \quad (34) \]
With (31) and (34), we get (35) which introduces:
- the contribution of the modes \( \neq i \) (\( H^r\text{psd}(k)k \)) (36),
- the contribution of the mode \( i \) alone (\( H^r\text{psd}(i) \)) (37)
and
- the cross terms (\( H^r\text{psd}(i,k) \)) (38):
\[ H^r\text{psd} = H^r\text{psd}(k)_{k \neq i} + H^r\text{psd}(i) + H^r\text{psd}(i,k) \quad (35) \]
\[ H^r\text{psd}(k)_{k \neq i} = (Mr.J + \sum_{k \neq i} Tk.Mk).\overline{(Mr.J + \sum_{k \neq i} Tk.Mk)} \quad (36) \]
\[ H^r\text{psd}(k)_{k \neq i} = Mr^2.J + \sum_{k \neq i} Mk^2 Tk.Tk' \]
\[ + 2.Mr.\sum_{k \neq i} Mk.ak \quad (36.1) \]
\[ + 2.\sum_{k \neq i} Mk.Mk'.ak.ak' \]
\[ + 2.\sum_{k \neq i} Mk.Mk'.bk bk' \]
\[ H^r\text{psd}(i) = M^2 Ti.Ti' \quad (37) \]
\[ H^r\text{psd}(i,k) = 2.(Mr + \sum_{k \neq i} Mk.ak).Mi.ai \]
\[ + 2.\sum_{k \neq i} Mk.Mi bk bi \quad (38) \]
with the coefficients \( ak \) and \( bk \) corresponding to reel and imaginary components of \( Tk \) (39):
\[ ak = \|Tk\| \cos(\theta k) \]
\[ bk = \|Tk\| \sin(\theta k) \quad (39) \]
with \( \theta k \) which varies from 0 at 0Hz to \(-\pi\) at \(+\infty\)Hz and nearly \(-\pi/2\) at \( \omega k \) (40)
\[ Tk(ak) = 1 - \frac{i}{2.\omega k} \quad (40) \]
\[ H^r\text{psd}(i) \]
In equation (38), the first term is a cosine term which equals to a constant value \( M0 \) (41) at 0Hz, and null at \(+\infty\)Hz as \( \|Tk\| \) tends to 0 at \(+\infty\)Hz.
\[ M0 = 2.(Mr + \sum_{k \neq i} Mk).Mi = 2.(M_{\text{tot}} - Mi).Mi \quad (41) \]
with \( M_{\text{tot}} \) the total mass of the equipment.
In equation (38), the \( 2^{nd} \) term is a sine term which is null either at 0Hz or at \(+\infty\)Hz.
These terms cannot be precisely identified as we do not know precisely all the modal parameters of the system.
To evaluate them, we can propose a method based on three steps:

**Step 1 a first level of approximation**

First approximation (42) of equation (38):

\[ H^r\text{psd}_1(i,k) = \text{term}^{\text{ci}}(i, \omega)\text{Mi}||T||\cos^2(\theta) \]
\[ + \text{term}^{\text{ci}}(i, \omega)\text{Mi}||T||\sin^2(\theta) \quad (42) \]
with
\[ \text{term}^{\text{ci}}(i, \omega) = 2.(M_{\text{tot}} - Mi) \]
\[ I_{(0 \rightarrow \omega k)} = 1 \text{ between } 0 \text{ and } \omega k \text{ and } 0 \text{ after } \omega k \text{ (to take into account the (41) term).} \]
The square sine or cosine in the approximation terms take into account the cross terms “\( \cos(\theta k) \cdot \cos(\theta i) \)” and “\( \sin(\theta k) \cdot \sin(\theta i) \)” in “\( ak.ai \)” and “\( bk.bi \)” respectively in equation (38)
Equation (35) becomes (44):
\[ H^r\text{psd}_1(k)_{k \neq i} = H^r\text{psd} - H^r\text{psd}(i) - H^r\text{psd}_1(i,k) \quad (44) \]
Equation (44) often over evaluates \( H^r\text{psd}(k)_{k \neq i} \) as the contribution of the cross terms \( H^r\text{psd}(i,k) \) are often under evaluated.
To correct this approximation we will perform a second calculation:

**Step 2 a second level of approximation**

Second approximation (45) of equation (38), from (44) and (36):
\[ H' \text{psd}_d(i, k) = \text{term}_1(i) \cdot M_i \| \mathbf{T} \| \cos^2(\theta) \] (45) \\
\[ + \text{term}_2(i) \cdot M_i \| \mathbf{T} \| \sin^2(\theta) \] \\
with \text{term}_2(i) = 2 \sqrt{H' \text{psd}_c(k,i)} \] (46)

The term \[ \sqrt{H' \text{psd}_c(k,i)} \] takes into account the term \[ M_r + \sum_{k \neq i} M_k a_k \] in the cosine term and the term \[ \sum_{k \neq i} M_k b_k \] in the sine term of equation (38).

As for step 1, the square sine or cosine in the approximation terms take into account the cross terms “\cos(\theta_k) \cdot \cos(\theta_i)” and “\sin(\theta_k) \cdot \sin(\theta_i)” in “\text{ak.ai}” and “\text{bk.bi}” respectively in equation (38).

When \[ H' \text{psd}(k,i) \] coming from equation (44) was over evaluated, the cosine and sine terms of equations (45) and (46) are this time over evaluated.

**Step 3 average value of cosine and sine terms**

The best approximation is to make an average of the cosine and sine terms coming from step 1 (42) and (43) and step 2 (45) and (46). We obtain then (47) for the cross terms \[ H' \text{psd}(i,k) \] of equation (38):

\[ H' \text{psd}(i,k) = \frac{1}{2} (\text{term}_1(i) + \text{term}_2(i)) \cdot M_i \| \mathbf{T} \| \cos^2(\theta) \] (47) \\
\[ + \frac{1}{2} (\text{term}_1(i) + \text{term}_2(i)) \cdot M_i \| \mathbf{T} \| \sin^2(\theta) \]

This introduces 2 cross terms to take into account:

\[ \text{termc} = \frac{1}{2} (\text{term}_1(i) + \text{term}_2(i)) \] (47.1) \\
\[ \text{terms} = \frac{1}{2} (\text{term}_1(i) + \text{term}_2(i)) \]

Finally, we can propose the estimation of \[ H' \text{psd}(k,i) \] (48):

\[ H' \text{psd}(k,i) = H' \text{psd} - H' \text{psd}(i) - H' \text{psd}(i,k) \] (48)

**Validation of the method:**

The accuracy of the method is validated by a comparison on different examples between an analytical evaluation of the contribution of the modes \[ k \neq i \] (\[ H' \text{psd}(k,i) \]) and the cross terms (\[ H' \text{psd}(i,k) \]) to the estimated values coming from the above method.

This example (table 1 and figure 4) is specifically defined to test the different possibilities:

- 2 large effective mass modes in the low frequency range and quite the same frequency (mode 2 with 4.4 kg at 146.59Hz and mode 3 with 4.82 kg at 147.04Hz),
- 1 large effective mass in the middle frequency range (mode 6 with 4 kg at 223.34Hz)
- 1 large effective mass in the high frequency range (mode 12 with 3.11 kg at 245.25Hz and
- a significant residual mass (2.5 kg)

<table>
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<th>qsi</th>
<th>freq</th>
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<td>0.02</td>
<td>147.04</td>
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**Mode 2 analysis (146.59Hz)**

On mode 2, even if the sine contributions with the higher modes are underestimated (figure 6), the global contribution of mode 2 \[ H' \text{psd}(i,k) \] (figure 8) and the global contribution of modes \[ \neq 2 \] \[ H' \text{psd}(k,i) \] (figure 7) are very well estimated.

This can be checked by the comparison between the global force PSD transfer function restitution from the mode 2 contribution evaluations and analytical (true value) calculation (figure 9). The difference appears only in the valleys.
Mode 3 analysis (147.04Hz)

The global contribution of mode 3 $H' \text{psd}(i,k)$ (figure 10) and the global contribution of modes $\neq 3 H' \text{psd}(k)_{i\neq i}$ (figure 11) are very well estimated.

As on mode 2, this can be checked by the comparison between the global force PSD transfer function restitution from the mode 3 contribution evaluations and analytical calculation (figure 12).
Mode 6 analysis (223.34Hz)

The same results can be obtained on mode 6.

Mode 12 analysis (245.25Hz)

The same results can be obtained on mode 12.
Conclusion: This method gives a very good estimation of the contribution of large effective mass modes, whatever the position of the mode in the modal analysis.

The method over evaluates the contribution of the modes in the valleys of the global interface force, as seen in the different HRPSD global restitution comparison figures. This has to be considered but will have only low impact due to the low levels in the valleys compared to the PSD peaks.

3.4 Translation of modal participation of Reaction PSD and proposed input

For each of the system modes which are to be translated we have defined a maximum of 3 equipment modes ie1 to ie3 (figure 3) on which we will translate the contribution of the system mode.

From the estimated PSD transfer functions calculated by the explained method ((48) (37) and (47)) and Uj"psd PSD of equipment average interface acceleration at system level we define the global reaction PSD on the equipment at system level by its different contributions:
- the contribution of the modes ≠i H′psd(k)psd, Uj"psd
- the contribution of the mode i alone H′psd(i)psd, Uj"psd
- and the cross terms H′psd(i,k)psd.

The contribution of the modes ≠i will be kept and 3 (maximum) contributions at equipment level will be calculated:
- Rjijpsd(ie1) + Rjijpsd(ie1,k) for equipment mode ie1,
- Rjijpsd(ie2) + Rjijpsd(ie2,k) for equipment mode ie2,
- Rjijpsd(ie3) + Rjijpsd(ie3,k) for equipment mode ie3,
with the notation Rjijpsd(ie) and Rjijpsd(ie,k) corresponding to the translated contribution on an equipment mode ie of the contributions of the system mode i.

The global reaction at equipment level will be defined as (49):

\[
\text{Rjijpsd}(i) = H^{'psd}(k)_{psd}Uj''psd + \frac{M(i,1)^2}{\sum M(ie)^2} (Rjijpsd(ie1) + Rjijpsd(ie1,k))
\]

(49)

\[
+ \frac{M(i,2)^2}{\sum M(ie)^2} (Rjijpsd(ie2) + Rjijpsd(ie2,k))
\]

\[
+ \frac{M(i,3)^2}{\sum M(ie)^2} (Rjijpsd(ie3) + Rjijpsd(ie3,k))
\]

Proposed input at the base of the equipment Uj"psde will be (50) calculated by the “Force limit method”:

\[
Uj''psde = Rjijpsd / H^{'psd}psd
\]

(50)

with H′psdpsd corresponding to the reaction transfer function of equipment in embedded configuration.

translation of the Rjijpsd(i) “diagonal” contribution

According to (18) and (37), the term to translate is the result of the product (51)

\[
Rjijpsd(i) = H^{'psd}(i)Uj''psd = Mpsd.Ti.Ti'*Uj''psd
\]

(51)

1) This contribution is translated first by an homothesis on the frequencies of the (51) terms (change of the frequency scale):

\[
freq(Rjijpsd(i)) = freq(Rjijpsd(i)).Freq(ie) / Freq(is)
\]

(52)

with Freq(ie) the frequency of the ie mode at equipment level and Freq(is) the frequency of the is mode at system level.

2) Then an homothesis on the level of the new contribution is to be performed to correct the RMS evolution of the contribution due to the frequency homothesis. But to prevent a change of the static contribution of this term, Hpsd(i) is first separated into 2 terms (53),

\[
H^{'psd}(i) = M^2.I_{\omega(k,ω(k))} + M^2.(Ti.Ti' - I_{\omega(k,ω(k))})
\]

(52)

with I_{\omega(k,ω(k))} = 1 between 0 and ω(k) and 0 after ω(k)
and the homothesis is only performed on the second term (53)

\[ \text{Rj} \text{psd}(\text{ie}) = M_\text{ie}^2 \text{I}_{(\text{ie},\text{ie})} \text{Lj}^\text{psd} \]

+ \( \frac{\text{Freq}(\text{ie})}{\text{Freq}(\text{ie})} \text{M}_{\text{ie}}^2 (\text{Ti, Ti}) \text{I}_{(\text{ie},\text{ie})} \text{Lj}^\text{psd} \)

(53)

**translation of the Rjpsd(i,k) “cross” contribution**

The new cross contribution (54) is evaluated by the two cross terms “termc” and “terms” (47.1).

\[ Rj\text{psd}(\text{ie}, k) = (\text{termc.Mie.Tie}) \cos^2(\theta\text{ie}) \]

+ terms.Mie.Tie sin^2(\theta\text{ie}))Lj^\text{psd} \]

(54)

with Mie the effective mass at equipment level (ie mode) and Tie the transmissivity factor calculated for the ie mode (same for \( \theta \text{ie} \)).

4. **APPLICATION ON AN EXAMPLE**

4.1 **Description of the example**

The example is a laser source developed by EADS/SODERN (with a suspended optical bench) integrated in a payload submitted to random environment, inspired from the development of the PHARAO/ACES (Atomic Clock Ensemble in Space), COLOMBUS payload.

![Figure 19 – artistic view of Laser Source](image)

The finite element model of this equipment is shown by figure 20:

![Figure 20 – detailed model of Laser Source](image)

This model is first reduced (figure 20.1) and integrated on a honeycomb plate with other equipments represented by constant masses linked by rigid elements (figure 21).

![Figure 20.1 – reduced model of Laser Source](image)

Nota: in the figures 22-23, 28-29 and 34-35, effective masses are normalized by the total mass of the Laser Source.
4.2 X axis analysis

figure 22-1 – X axis identification process result on embedded model of laser source

<table>
<thead>
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<th>Mode</th>
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<th>Y</th>
<th>Z</th>
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<th>Y</th>
<th>Z</th>
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figure 22-2 – X axis identification process result on embedded model of laser source

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figure 23-1 – X axis identification process result on model of Laser Source at system level

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figure 23-2 – X axis identification process result on model of Laser Source at system level

Figure 24 - comparison of Laser Source X transfer functions between equipment and system level (global force + optical bench acceleration)

Figure 25 - comparison of X modes 1 and 2 PSD contributions before and after translation process

Figure 26 - comparison of X global interface force PSD before and after translation process
Figure 27 - comparison of X interface acceleration PSD to “force limit method” acceleration PSD with and without translation process

force limit method acceleration PSD without translation process
force limit method acceleration PSD with translation process

X axis conclusion: The automatic primary notching proposed at 147.6Hz by the “force limit method” without translation process is not justified. “force limit method” with translation process proposes an input > average acceleration -> no notching allowed

- addition of 2 system mode contributions
- lateral modes of optical bench at system level don’t create lateral deformation of interface base plate (honeycomb).

4.3 Y axis analysis

Figure 27.1 - Zoom on figure 27

figure 28-1 – Y axis identification process result on embedded model of laser source

<table>
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<th>Mode</th>
<th>Frequency</th>
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<th>AcceY</th>
<th>Qxi</th>
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<th>TransEffY</th>
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figure 28-2 – Y axis identification process result on model of Laser Source at system level

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<th>AcceY</th>
<th>Qxi</th>
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figure 29-1 – Y axis identification process result on model of Laser Source at system level

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</tr>
<tr>
<td>21 Y</td>
<td>324.8</td>
<td>0.0245959</td>
<td>0.108693</td>
<td>0</td>
<td>0</td>
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</tr>
<tr>
<td>24 Y</td>
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<td>0.0328216</td>
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<tr>
<td>29 Y</td>
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<td>0.0526194</td>
<td>0.424379</td>
<td>0</td>
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<td>0</td>
</tr>
<tr>
<td>33 Y</td>
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<td>0.0453497</td>
<td>-0.224949</td>
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<td>0</td>
<td>0</td>
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<tr>
<td>37 Y</td>
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</tr>
<tr>
<td>39 Y</td>
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<td>0.1</td>
<td>0.0749915</td>
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</tr>
</tbody>
</table>

Translation request
Figure 30 - comparison of Laser Source Y transfer functions between equipment and system level (global force + optical bench acceleration)

Figure 31 - comparison of Y modes 1 and 4 PSD contributions before and after translation process

Figure 32 - comparison of Y global interface force PSD before and after translation process

Figure 33 - comparison of Y interface acceleration PSD to “force limit method” acceleration PSD with and without translation process

Figure 33.1 - Zoom on figure 33

Y axis conclusion: As for X axis, the automatic primary notching proposed at 146.9Hz by the “force limit method” without translation process is not justified. The “force limit method” with translation process proposes an input \( \approx \) average acceleration -> little notching allowed reasons:
- addition of 2 system mode contributions
- lateral modes of optical bench at system level don’t create lateral deformation of interface base plate (honeycomb).
### 4.4 Z axis analysis

#### Table 34-1

<table>
<thead>
<tr>
<th>Mode</th>
<th>Equipment</th>
<th>System level</th>
<th>Mode</th>
<th>Equipment</th>
<th>System level</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 mode</td>
<td>Z</td>
<td>capot_sup2Z</td>
<td>0,02124</td>
<td>Z</td>
<td>0,00588901</td>
</tr>
<tr>
<td>4 mode</td>
<td>Z</td>
<td>milieu_bench</td>
<td>0,07767</td>
<td>Z</td>
<td>1,05633</td>
</tr>
<tr>
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<td>capot_sup1Z</td>
<td>0,04583</td>
<td>Z</td>
<td>0,0209586</td>
</tr>
<tr>
<td>11 mode</td>
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<td>capot_sup1Z</td>
<td>0,02776</td>
<td>Z</td>
<td>0,0407515</td>
</tr>
<tr>
<td>15 mode</td>
<td>Z</td>
<td>capot_sup1Z</td>
<td>0,01750</td>
<td>Z</td>
<td>0,0385076</td>
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<tr>
<td>18 mode</td>
<td>Z</td>
<td>interf_bench</td>
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<td>Z</td>
<td>-0,00504017</td>
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<tr>
<td>19 mode</td>
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<td>capot_sup1Z</td>
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<tr>
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<td>interf_bench</td>
<td>0,02092</td>
<td>Z</td>
<td>0,0839223</td>
</tr>
</tbody>
</table>

#### Table 35-1

<table>
<thead>
<tr>
<th>Mode</th>
<th>Equipment</th>
<th>System level</th>
<th>Mode</th>
<th>Equipment</th>
<th>System level</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 mode</td>
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<td>0,01741</td>
<td>Z</td>
<td>-0,296145</td>
</tr>
<tr>
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<td>Z</td>
<td>capot_sup3Z</td>
<td>0,03447</td>
<td>Z</td>
<td>-0,351296</td>
</tr>
<tr>
<td>8 mode</td>
<td>Z</td>
<td>capot_sup3Z</td>
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<td>Z</td>
<td>-0,296851</td>
</tr>
<tr>
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<td>milieu_bench</td>
<td>0,07446</td>
<td>Z</td>
<td>1,05495</td>
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<tr>
<td>13 mode</td>
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<td>0,0241834</td>
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<td>Z</td>
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<tr>
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<td>capot_sup1Z</td>
<td>0,01737</td>
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<tr>
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<td>milieu_bench</td>
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<td>35 mode</td>
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<tr>
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<td>0,0373372</td>
</tr>
</tbody>
</table>

**Figure 34-1** – Z axis identification process result on embedded model of laser source

**Figure 34-2** – Z axis identification process result on embedded model of laser source

**Figure 35-1** – Z axis identification process result on model of Laser Source at system level

**Figure 35-2** – Z axis identification process result on model of Laser Source at system level

**Figure 36** – Comparison of Laser Source Z transfer functions between equipment and system level (global force + optical bench acceleration)

**Figure 37** – Comparison of Z mode 4 PSD contribution before and after translation process

**Figure 38** – Comparison of Z global interface force PSD before and after translation process
Figure 39 - comparison of Z interface acceleration PSD to “force limit method” acceleration PSD with and without translation process

System and equipment modes

Figure 39.1 - Zoom on figure 39

Z axis conclusion: On this axis, the Z mode at 166Hz at system level has been shifted at 168.5Hz at equipment level which slightly changes the input with or without the translation process -> no notching allowed

reason: - little coupling between vertical mode of optical bench at system level and interface base plate (honeycomb).

5. CONCLUSION AND PERSPECTIVES

This paper proposes an additional tool which can efficiently help the system level mechanical engineer who has to define equipment random specification and to evaluate the depth of allowable primary notchings in random.

This method is based only on mechanical analyses, but could be adapted for low level sine test results, with a function estimating the global force at COM of the equipment (defined and checked with the FEM model, for instance based on accelerometer measurements multiplied by mass coefficients).

During mechanical tests, primary notching could also be controlled by PSD limitations on well placed accelerometers on the structure.

REFERENCES

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4. NASA technical handbook “Force limited vibration testing” NASA-HDBK-P011 April 1, 1999